TURBULENT MOTION OF AIR MIXTURES IN PIPES

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The additional resistance due to dissipation of the energy of turbulent fluctuations by solid particles is determined on the basis of a theoretical analysis. The results obtained are compared with experimental data.

In calculating the characteristics of pipe flows of air mixtures with dispersed solids, it is of considerable importance to calculate with sufficient accuracy the additional resistances due to the presence of the solid particles in the flow.

In this paper an attempt is made to determine the importance of the effect on additional resistance of the dissipation of the energy of turbulent fluctuations by fine particles (diameter less than 10^{-4} m).

We shall examine the motion of a highly dispersed air mixture at considerable velocities, making the assumptions, usual for fine particles, that the mean translational velocities of the particles and the carrier medium are equal, and that the distribution of particles over the cross section of the flow is fairly uniform. We shall also assume that the influence of the solids on the kinematic structure of the flow is negligible (low concentrations), and that the number of collisions between particles is small. The question of the influence of the gravitational field on the motion of the particles will be examined below.

When the particles move in large-scale eddies with low wave numbers (low frequencies), entrainment is complete. In this case there is additional expenditure of energy due to enhanced momentum transfer as a result of the increase in the density of the medium, i.e., the physical picture is one of transport of a fluid of high specific weight with a viscosity equal to that of the carrier medium.

When the motion is in small-scale eddies with high wave numbers (high frequencies), the entrainment of the particles is incomplete, and there is relative motion between the particles and the medium, resulting in additional dissipation of the turbulent fluctuation energy.

We shall select some micro-volume of the fluid, moving with mean velocity $\bar{u}$, the fluctuation velocity component u_i' (i = 1; 2; 3) being a function of some characteristic frequency ω associated with the scale of the chosen volume. We shall examine the motion of a particle in this volume, in a coordinate system moving with velocity $\bar{u}$.

For nonuniform motion of a particle in a fluid in the general case, in addition to viscous resistance, we should take into account the resistance due to the expenditure of energy in accelerating the fluid itself. Fuchs [1] showed that for nonuniform motion of a solid particle in a gas, when $\rho_s \gg \rho_0$, the unsteady effects may be neglected at small Re_*, and the resistance may be considered to be noninertial, i.e., to correspond to the resistance at a constant velocity equal to the velocity at the given instant. Therefore, for finely dispersed air mixtures with spherical particles of diameter $<10^{-4}$ m, the resistance of the medium when the particle motion is nonuniform is given with sufficient accuracy by Stokes law. Because the density of the medium, ρ_0, is small, we may consider that $\rho_s - \rho_0 \approx \rho_s$.

Then in the chosen coordinate system, with gravity taken into account, the equation of motion of a particle may be written in the form

$$\rho_s V \frac{dw_i}{dt} = -3\pi d \eta w_i + \sigma_i g \rho_s V, \qquad (1)$$

where

$$\sigma_1 = \sigma_3 = 0; \ \sigma_2 = 1.$$

Bearing in mind that $v_i = u_i + \omega_i$, we obtain

$$\frac{dw_i}{dt} + \frac{w_i}{\tau} = -\frac{du_i'}{dt} + \sigma_i g, \qquad (2)$$

where

$$\tau = \rho_s d^2/18\eta.$$

The quantity τ has the dimension of time and is the characteristic quantity determining the motion of the particle.

Because of the quasi-periodic nature of the turbulent fluctuations, for some specific perturbation scale u_i' may, as usual, be written approximately as

$$u_i' = u_{0i}' \sin \omega t. \qquad (3)$$

The solution of an equation of type (2) for the motion of a particle in a periodically fluctuating fluid (but without allowance for the field of gravity) was analyzed in detail in [1]. By a similar method, we can find ω_i for steady motion from (2) and (3).

For the plane problem (i = 1; 2), the components of relative velocity of the particle may be written as follows:

for the longitudinal component

$$w_1 = -\frac{u_{01}' \omega\tau}{\sqrt{1+\omega^2\tau^2}} \sin\left(\omega t + \frac{\pi}{2} - \varphi\right), \qquad (4)$$

for the vertical component

$$w_2 = -\frac{u_{02}' \omega\tau}{\sqrt{1+\omega^2\tau^2}} \sin\left(\omega t + \frac{\pi}{2} - \varphi\right) + g\tau, \qquad (5)$$

where

$$\varphi = \operatorname{arc\,tg} \omega\tau.$$

It should be noted that $g\tau = \omega_*$.

We shall examine the conditions under which the force of gravity may be neglected. We find the mean square of the first term on the right of Eq. (5)

$$\left[\frac{1}{T}\int_0^T \frac{(u'_{02})^2\omega^2\tau^2}{1+\omega^2\tau^2}\sin^2\left(\omega t + \frac{\pi}{2} - \varphi\right)\right]^{1/2} = \frac{u'_{02}}{\sqrt{2}}\,\frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}.$$

In addition, we similarly obtain from (3)

$$\overline{u'_i} = u'_{0i}/\sqrt{2}. \quad (6)$$

The region in which gravity forces may be neglected is defined by the inequality

$$\overline{u'_2}\omega/\sqrt{1+\omega^2\tau^2} \gg g. \quad (7)$$

At high frequencies when $\omega^2\tau^2 \gg 1$, condition (7) may be written in the form

$$\overline{u'_2}/\tau \gg g, \quad (8)$$

and at low frequencies when $\omega^2\tau^2 \ll 1$ in the form

$$\overline{u'_2}\omega \gg g. \quad (9)$$

In pneumatic conveyor systems, the transport velocity is usually assumed to be about 20–25 m/sec. The intensity of turbulence is roughly 0.04. Therefore the mean fluctuation velocity in these conditions may be taken to be $\overline{u}' = 1$ m/sec.

For particles of diameter 10^{-4} to 10^{-5} m and density $\rho_s = 1000$ kg/m^3, τ varies in the range $3 \cdot 10^{-2}$ to $3 \cdot 10^{-4}$ sec.

Condition (8) is satisfied with sufficient accuracy when $\tau < 3 \cdot 10^{-2}$ sec. Condition (9) is satisfied when $\omega > 50$ cps.

We shall examine the experimental characteristics for a turbulent flow presented in [2]. It can be seen from Fig. 4.3 of [2] that for the flow core the least wave numbers are of the order of $2 \cdot 10^{-2}$. Since the wave number is defined as $k = \omega/U_m$, and under the experimental conditions $U_m = 30$ m/sec, the least frequencies ω will correspondingly be about 60.

Therefore, when $d < 10^{-4}$ m, for an approximate solution of the problem, the influence of the gravity forces may be neglected, and both components of relative velocity of the particle will be given by (4).

When the particle moves in a viscous medium, the work of the resistance forces may be expressed as

$$\Omega_i = \int_0^x 3\pi d\eta v_i\,dx. \quad (10)$$

In the case examined, we have simultaneous motion of the particles and of the medium, and we must therefore replace v_i in (10) by ω_i. Since $dx = \omega_i dt$, we find the work done by the resistance forces in unit time, taking account of (4), to be

$$q'_i = 3\pi d\eta \frac{1}{T}\int_0^T w_i^2\,dt = \frac{3}{2}\pi d\eta (u'_{0i})^2 \frac{\omega^2\tau^2}{1+\omega^2\tau^2}. \quad (11)$$

A similar approach was used in [3] to estimate the dissipation of the energy of turbulent fluctuations by solids. However, the author did not demonstrate the relationship between this effect and the general parameters of the two-phase flow.

The power lost in particle friction in the velocity fluctuations in unit volume is

$$q_i = q'_i n. \quad (12)$$

From (11) and (12) we obtain an equation for the total energy dissipated by particle friction in longitudinal and vertical fluctuations in unit volume and unit time:

$$q = \frac{9\eta s}{d^2}\,\frac{\omega^2\tau^2}{1+\omega^2\tau^2}\left[(u'_{01})^2 + (u'_{02})^2\right]. \quad (13)$$

Since the turbulence represents the superposition of eddies of different scales, particles moving in large-scale eddies simultaneously take part in the motion of small and very small eddies located within the large eddies. The minimum scale of eddies, or the internal scale of turbulence, has a dimension of the order 10^{-2} to 10^{-3} m, which is 10–100 times greater than the size of the particles, so that the main mass of particles interacts with precisely the micro-eddies of the fluid; and the dissipation of fluctuation energy is determined by this interaction. A small fraction of the particles at the junction between large eddies takes part directly

Total Energy Losses in the Motion of Air Mixtures and Losses Due to the Dissipation of Fluctuation Energy

Results of [4]					Calculated values $\Delta i \cdot 10^{-2}$, N · m^{-3}	
U_m, m · sec^{-1}	μ	$i_s \cdot 10^{-2}$, N · m^{-3}	$i_0 \cdot 10^{-2}$, N · m^{-3}	$(i_s - i_0) \cdot 10^{-2}$, N · m^{-3}	$k > k_1$	$k > k_2$
15	63	8.7	0.49	8.21	0.085	0.389
20	47	9.6	0.78	8.92		
25	38	10.3	1.27	9.03		
30	31.5	11.1	1.67	9.43		

in the large-scale fluctuations, but the energy dissipated in this way is negligible, since in this case $\omega \to 0$, $w_i \to 0$, and $q \to 0$.

Because of the statistical nature of turbulence, it is impossible to distinguish any particular size of micro-eddy interacting with the particles, or the corresponding frequency. It may be assumed, however, that energy dissipation occurs in the interaction of particles with small eddies in the region of high wave numbers, in excess of some characteristic number k_1. In the high wave number region Kolmogorov's hypotheses are confirmed:

$$E(k) \sim k^{-5/3}.$$

The mean fluctuation velocity component in the region $k > k_1$ may be found from the equation

$$\overline{(u_i')}^2 = \int_{k_1}^{\infty} Ak^{-5/3}\,dk, \tag{14}$$

where

$$A = \text{const}.$$

We shall use the experimental data given in [2] (pipe diameter 0.247 m, U_m = 30 m/sec). We choose the point $k_1 = 10$ on Fig. 4.3 of [2] as the lower boundary of the region of intense dissipation of eddies by particles. It should be noted that, since the mean fluctuating velocity is determined over the whole region above k_1, some arbitrariness in the choice of this point has little influence on the result.

For comparison we choose the point $k_2 = 10^0$, which includes the region of both micro-eddies and average energy-containing eddies; this gives a deliberately overestimated result for the energy dissipation. The corresponding frequencies will be

$$\omega_1 = k_1U_m = 10\cdot 3\cdot 10^3 = 3\cdot 10^4\,\text{sec}^{-1};\ z_1 = 4.77\cdot 10^3\,\text{sec}^{-1};$$
$$\omega_2 = 1\cdot 3\cdot 10^3 = 3\cdot 10^3\,\text{sec}^{-1};\ z_2 = 4.77\cdot 10^2\,\text{sec}^{-1}.$$

We note that for particles of diameter $2.5\cdot 10^{-5}$ m and $\tau = 1.93\cdot 10^{-3}$ sec, even with $\omega = \omega_2 = 3\cdot 10^3$ sec^{-1}, $\omega^2\tau^2 \gg 1$.

Therefore, for the region examined, with sufficient accuracy it may be assumed that

$$\omega^2\tau^2/(1+\omega^2\tau^2) \approx 1. \tag{15}$$

Since the turbulence is isotropic in the high wave number region, replacing the peak values of the velocity fluctuations by their mean square values according to (6) and taking (15) into account, we may write (13) in the form

$$q = 36a^2(\eta/d^2)\mu(\rho_0\rho_s)U_m^2, \tag{16}$$

or

$$q = 2a^2\mu U_m^2\rho_0/\tau, \tag{17}$$

where a is defined in the region $k > k_1$.

We shall examine a section of pipe with cross-sectional area F and length l. We denote by dp the pressure drop in a length dl due to dissipation of fluctuation energy by the particles. The work done by the pressure forces will be equal to $Fdp\,dl$.

The work done by the dissipation forces is

$$qFdldt = qFdl\,dl/U_m.$$

Hence we obtain

$$dp = qdl/U_m;\quad p_1 - p_2 = ql/U_m.$$

Taking into account that $(p_1 - p_2)/l = i$, we finally obtain

$$\Delta i = 36a^2(\eta/d^2)\mu(\rho_0/\rho_s)U_m, \tag{18}$$

or

$$\Delta i = 2a^2\mu(\rho_0/\tau)U_m. \tag{19}$$

To evaluate the influence of the dissipation of fluctuation energy by the particles, calculate Δi from (18) and compare this with experimental data on the pneumatic transport of coal dust in a vertical pipe [4]. From Fig. 4.3 of [2] we determine, in accordance with (14), the mean square fluctuation velocities for regions $k > k_1$ and $k > k_2$. The squares of the intensities of turbulence are

$$a_1^2 = 0.423\cdot 10^{-4},\quad a_2^2 = 0.193\cdot 10^{-3}.$$

In [4] the density of the coal was 1660 kg/m^3 and the mean square particle diameter was $4.74\cdot 10^{-5}$ m. The results of the experiment (without allowance for the weight of the column of air mixture) and the calculated values of Δi are given in the table.

It can be seen from the table that the additional flow resistance due to dissipation of fluctuation energy by the particles, even for the region $k > k_2$, i.e., when clearly overestimated, constitutes only about 4% of the additional resistance due to the presence of solids in the stream. Therefore, in practical calculations this particular effect need not be taken into account.

NOTATION

$\overline{u}$) mean velocity of fluid; u_i' (i = 1, 2, 3)) fluctuating component of velocity; ω) angular frequency; ρ_s and ρ_0) density of solids and of air, respectively; Re_*) Reynolds number for particle; v_i) particle velocity; w_i) relative particle velocity; V) particle volume; d) particle diameter; η) viscosity of air; g) acceleration due to gravity; τ) particle relaxation time; u_{0i}) amplitude of turbulent velocity fluctuations; z) frequency of fluctuations; t) time; φ) phase shift angle; w_*) hydraulic size of particles; $\overline{u_i'}$) mean square velocity of turbulent fluctuations; U_m) mean flow velocity; k) wave number; Ω_i) work done by viscosity forces; q_i', q_i, q) power; $T = 2\pi/\omega$) period of oscillation; $n = 6s/\pi d^3$) number of particles in unit volume; s) volume concentration of air mixture; E(k)) spectral function of energy of turbulent fluctuations; $a = \overline{u}/U_m$) intensity of turbulence; F) area of pipe cross section of air mixture; E(k) spectral function of energy of turbulent fluctuations; $a = \overline{u}'/U_m$) intensity of turbulence; F) area of pipe cross section; l) length of pipe; p) pressure; i) specific pressure loss (hydraulic gradient); Δi) specific pressure loss due to dissipation of fluctuation energy by particles; μ) mass concentration of air mixture.

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